Western Cape Government

Education

Western Cape Education Department

Telematics Learning resource

First term 2015

MATHEMATICS

GRADE 12

First term 2015 **February - March (Grade 12)**				
Day	**Date**	**Time**	**Subject**	**Topic**
Wednesday	18 February	15:00 – 16:00	Mathematics	Inverses of functions: Inverses of $y=mx+c$ and $y=ax^2$
		16:00 – 17:00	Wiskunde	Inverses van functions: Inverses van $y=mx+c$ en $y=ax^2$
Thursday	19 March	15:00 – 16:00	Mathematics	The log and exponential functions as inverses of each other
		16:00 – 17:00	Accounting	Companies: Preparation of Cash Flow Statement
Friday	20 March	15:00 – 16:00	Wiskunde	Die log- en eksponesiale functions as inverses van mekaar
		16:00 – 17:00	Rekeningkunde	Maatskappye: Opstel van Kontantvloeistaat

Session :1 The concept of an inverse; the inverses of $y = mx + x$ en $y = ax^2$

An inverse function is a function which does the "reverse" of a given function. More formally, if f is a function with domain X, then f^{-1} is its inverse function if and only if $f^{-1}(f(x)) = x$ for every $x \in X$.

$y = f(x)$: indicates a function

$y_1 = f(x_1)$: indicates we must substitute a specific x_1 value into the function to get the corresponding y_1 value

$f^{-1}(y) = x$: indicates the inverse function

$f^{-1}(y_1) = x_1$: indicates we must substitute a specific y_1 value into the inverse to return the specific x_1 value

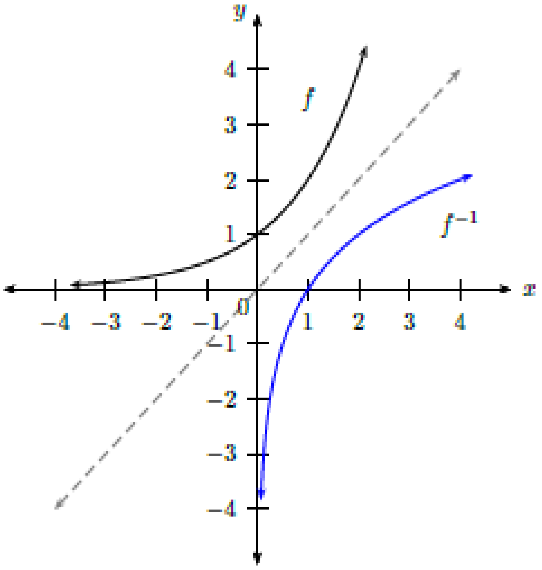
A function must be a one-to-one relation if its inverse is to be a function. If a function f has an inverse function f^{-1}, then f is said to be invertible.

Given the function $f(x)$, we determine the inverse $f^{-1}(x)$ by:

- interchanging x and y in the equation;
- making y the subject of the equation;
- expressing the new equation in function notation.

Note: if the inverse is not a function then it cannot be written in function notation. For example, the inverse of $f(x) = 3x^2$ cannot be written as $f^{-1}(x) = \pm\sqrt{\frac{1}{3}x}$ as it is not a function. We write the inverse as $y = \pm\sqrt{\frac{1}{3}x}$ and conclude that f is not invertible.

If we represent the function f and the inverse function f^{-1} graphically, the two graphs are reflected about the line $y = x$. Any point on the line $y = x$ has x- and y-coordinates with the same numerical value, for example $(-3; -3)$ and $(\frac{4}{5}; \frac{4}{5})$. Therefore interchanging the x- and y-values makes no difference.



Important: for f^{-1}, the superscript -1 is not an exponent. It is the notation for indicating the inverse of a function. Do not confuse this with exponents, such as $\left(\frac{1}{2}\right)^{-1}$ or $3+x^{-1}$.

Be careful not to confuse the inverse of a function and the reciprocal of a function:

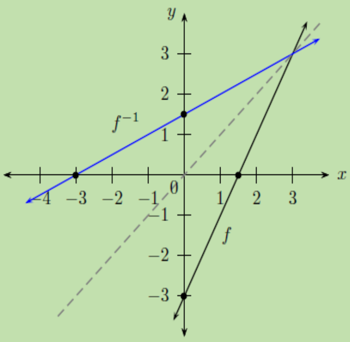
Inverse	Reciprocal
$f^{-1}(x)$	$[f(x)]^{-1} = \frac{1}{f(x)}$
$f(x)$ and $f^{-1}(x)$ symmetrical about $y = x$	$f(x) \times \frac{1}{f(x)} = 1$
Example:	Example:
$g(x) = 5x \therefore g^{-1}(x) = \frac{x}{5}$	$g(x) = 5x \therefore \frac{1}{g(x)} = \frac{1}{5x}$

Below is an example of the inverse of $y = mx + c$

$$\begin{aligned} &\text{Let } y = 2x - 3 \\ \text{Interchange } x \text{ and } y: \quad & x = 2y - 3 \\ & x + 3 = 2y \\ & \frac{1}{2}(x+3) = y \\ & \therefore y = \frac{x}{2} + \frac{3}{2} \end{aligned}$$

Therefore, $f^{-1}(x) = \frac{x}{2} + \frac{3}{2}$.

Step 2: Sketch the graphs on the same system of axes



The graph of $f^{-1}(x)$ is the reflection of $f(x)$ about the line $y = x$.

Please note that when we are dealing with the inverse of a parabola (quadratic function), we encounter the problem that the inverse is not a function. This is because the quadratic function is not a one-to-one relation (mapping). In order to ensure that we obtain a function for the inverse of the parabola, we must restrict the domain of the original function (n.b. **not** the inverse). See the example below:

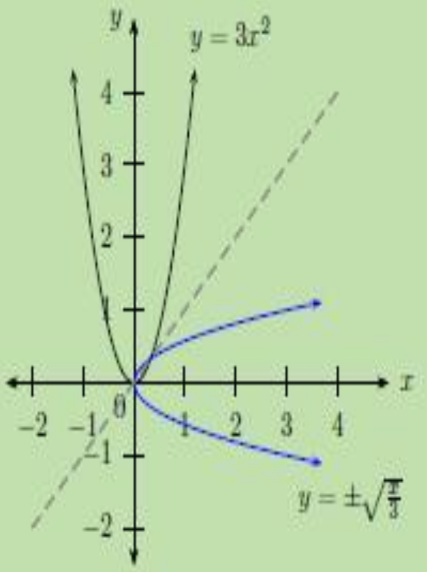
Let $y = 3x^2$

Interchange x and y: $x = 3y^2$

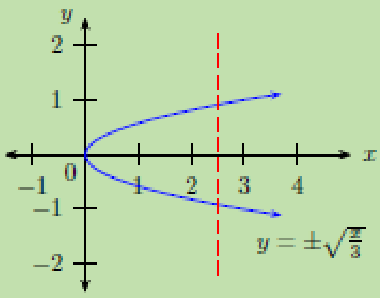
$$\frac{x}{3} = y^2$$

$$\therefore y = \pm\sqrt{\frac{x}{3}} \qquad (x \geq 0)$$

Step 2: Sketch the graphs on the same system of axes



Notice that the inverse does not pass the vertical line test and therefore is not a function.



To determine the inverse function of $y = ax^2$:

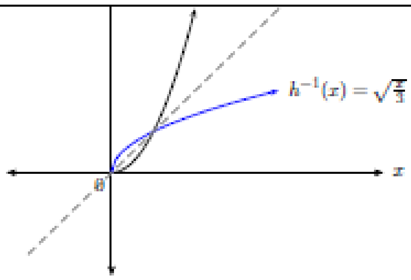
(1) Interchange x and y : $x = ay^2$

(2) Make y the subject of the equation : $\frac{x}{a} = y^2$

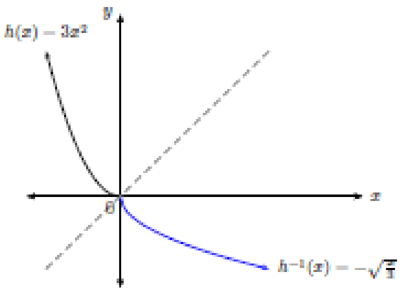
$\therefore y = \pm\sqrt{\frac{x}{a}} \quad (x \geq 0)$

The vertical line test shows that the inverse of a parabola is not a function. However, we can limit the domain of the parabola so that the inverse of the parabola is a function.

. We can do this in two ways as illustrated below:.In the first diagram we have restricted the domain to $x \geq 0$ and in the second diagram to: $x \leq 0$



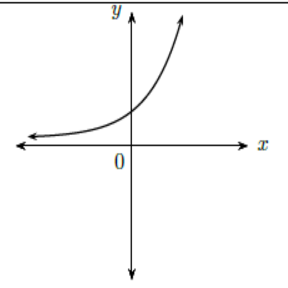
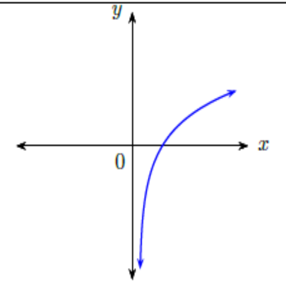
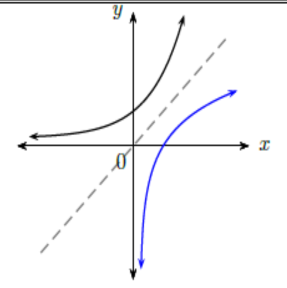
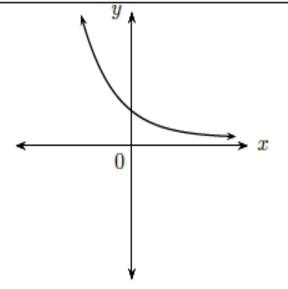
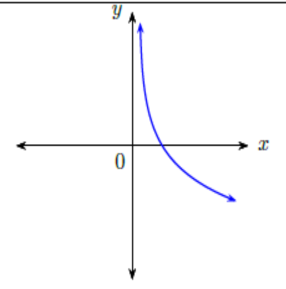
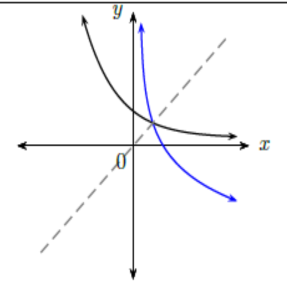
- If the restriction on the domain of h is $x \leq 0$, then $h^{-1}(x) = -\sqrt{\frac{x}{3}}$ would also be a function.



Session 2

The Log-function and its inverse:

Summary of graphs: $y = b^x$ **and** $y = \log_b x$

	Exponential function	Logarithmic function	As inverses on the same system of axes
	$y = b^x$	$y = \log_b x$	
$b > 1$			
$0 < b < 1$			

EXERCISES

The function concept

1.1 State if the following are true or false. Provide a reason for each answer.

1.1.1 The inverse of $f = \{(2;3);(4;7)\}$ is $\{(3:2);(7;4)\}$ (2)

1.1.2 $f = \{(2;-3);(4;6);(-2;-3);(6;4)\}$ is a many-to-one relation (2)

1.1.3 The inverse of 1.1.2 is a function (2)

1.1.4 The domain of 1.1.2 is $D = \{2;4;6\}$ (2)

1.1.5 The function f and its inverse f^{-1} are reflections in the line $y = -x$ (2)

The inverse of $y = mx + c$

1. Given $f(x) = 2x - 7$

   1.1 Is $f(x)$ a function? Explain your answer

   1.2 Write down the domain and range of $f(x)$

   1.3 Determine $f^{-1}(x)$

   1.4 Draw graphs of $f(x) and f^{-1}(x)$ on one system of axes

   1.5 Give the equation of the line of reflection between the two graphs and indicate this line on the graph using a broken line.

2. Given that $f^{-1}(x) = -2x + 4$, Determine $f(x)$

3. $f(x) = \frac{2}{3}x$ and $g(x) = -3x - 9$. Determine the point(s) of intersection of f^{-1} and g^{-1}.

The inverse of $y = ax^2$

1 Given the function $f(x) = x^2$.

1.1 Determine $f^{-1}(x)$. (3)

1.2 Draw the grafiek of $f^{-1}(x)$. (2)

1.3 Explain why $f^{-1}(x)$ will not be function? (1)

1.4 Explain how you will restrict the domain of $f(x)$ to ensure that $f^{-1}(x)$ will also be a function? (2)

2. Given $f(x) = \frac{1}{2}x^2$;.

2.1 Determine $die\ inverse\ van\ f(x)$ (3)

2.2 Is $the\ inverse\ van\ f(x)$ a function of nie? Gee 'n rede vir jou antwoord. (2)

2.3 How will you restrict the domain of the original function so as to ensure that $f^{-1}(x)$ will also be a function? (1)

2.4 Draw graphs of $f(x) and\ f^{-1}(x)$ on the same system of axes (3)

2.5 Determine the point(s) where $f(x) and\ f^{-1}(x)$ will intersect each other (4)

3 Given: $f(x) = -2x^2$

3.1 Explain why, if the domain of this function is not restricted, its inverse will not be a function?

3.2 Write down the equation of the inverse, $f^{-1}(x)$ of $f(x) = -2x^2$ for $x \in (-\infty;0]$ in the form $f^{-1}(x) = \ldots$

3.3 Write down the domain of $f^{-1}(x)$

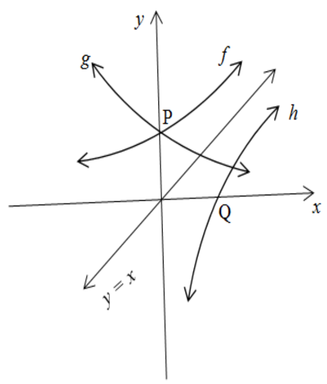
3.4 Draw the graphs of both $f(x) = -2x^2$ vir $x \in (-\infty;0]$ and $f^{-1}(x)$ on the same system of axes.

Log- and exponential functions as inverses of each other

2.2 The graph alongside shows the functions g, f and h.
f and g is symmetrical with respect to the y-axis
f and h is symmetrical with respect to the line
$y = x$. If $f(x) = a^x$ and the point (1 ; 4) lies on $f(x)$:

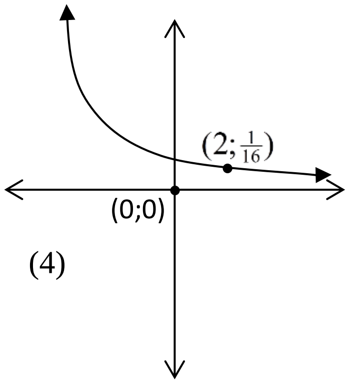
2.2.1 Determine the value of a *(2)*
2.2.2 Write down the coördinates of P and Q (2)
2.2.3 Write down the equations of g, h and g^{-1} (6)

(2)



6.1 The figure represents the graph of $f(x) = a^x$.
Calculate the value of a. (2)

6.2 Draw a graph of $k(x)$ if k is the inverse of f . Show the intercepts with the axes, as well as the coördinates of one other point. Also indicate the asymtotes (4)

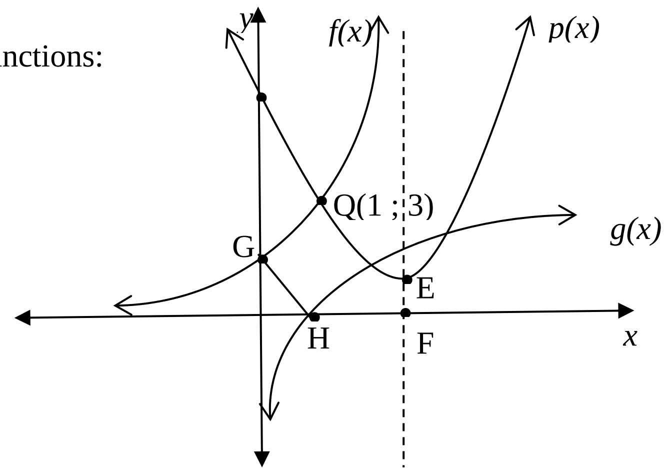


Question 2

2.1 The diagram alongside show the functions:

$f(x) = k^x$;
$p(x) = ax^2 + bx + c$
and $g(x) = log_m x$

The minimum value of the function $p(x)$ is equal to 1 where $x = 3$.
The turning point of the parabola is at the point F.
EF is parallel to the y-axis.



2.1.1 Determine the values of a, b, c, m and k. (5)
2.1.2 Calculate the length of EF and GH correct to two decimal places (3)
2.1.3 Determine the equations of $f^{-1}(x)$ and $g^{-1}(x)$ (4)
2.1.4 Hence explain why or why not, $f(x)$ and $g(x)$ will be symmetrical with respect to the line $y = x$ (2)

Western Cape Government

Education

TELEMATICS TEACHING PROJECT

GRADE 12

MATHEMATICS WORKBOOK

2015

Terms 2 and 3

DearGrade 12 Learner

In the 2nd term the presenters will revise differential calculas with specific focus on:

- The cubic graph
- Optimisation.

The lesson in Term 3 will focus on revision of Grade11 and Grade 12 geometry. The Grade 11 geometry entails the circle geometry theorems dealing with angles in a circle, cyclic quadrilaterals and tangents. The Grade 12 geometry is based on ratio and proportion as well as similar triangles. Grade 11 geometry is especially important with regard to similar triangles and hence this work must be thoroughly understood.

Your teacher should indicate to you exactly which theorems you have to study for examination purposes but no proofs of the inverses of these theorems will be examined.

This workbook provides the activities for these sessions. Please make sure that you bring this workbook along to each and every Telematics session.

At the start of each lesson, the presenters will provide you with a summary of the important concepts and together with you will work though the activities. You are encouraged to come prepared, have a pen and enough paper (ideally a hard cover exercise book) and your scientific calculator with you.

You are also encouraged to participate fully in each lesson by asking questions and working out the exercises, and where you are asked to do so, sms or e-mail your answers to the studio.

Schedule

Date	**Time**	**Subject**	**Topic**
Monday 11 May	15:00 – 16:00	Mathematics	Graphs of cubic functions
Monday 18 May	15:00 – 16:00	Wiskunde	Grafieke van derde graadse funksies
Monday 1 June	15:00 – 16:00	Mathematics	Applications of calculus: optimisation and rate of change
Wednesday 3 June	15:00 – 16:00	Wiskunde	Toepassings van differensiale rekene: Optimering en tempo van verandering
Thursday 27 August	15:00 – 16:00	Mathematics	Geometry similar triangles
Tuesday 1 September	15:00 – 16:00	Wiskunde	Meetkunde: gelykvormige driehoeke

Grade 12 calculus

Lesson 1: Graphs. In this lesson you will work through 3 types of questions regarding graphs
1: Drawing cubic graphs
2. Given the graphs, answer interpretive questions
3. Given the graphs of the derivative, answer interpretive questions

1A

Given: $f(x) = 2x^3 - x^2 - 4x + 3$

A1	Show that $(x - 1)$ is a factor of $f(x)$.	(2)
A2	Hence factorise $f(x)$ completely.	(2)
A3	Determine the co-ordinates of the turning points of f.	(4)
A4	Draw a neat sketch graph of f indicating the co-ordinates of the turning points as well as the x-intercepts.	(4)
A5	For which value of x will f have a point of inflection?	(4)
		[16]

1B

Given $f(x) = x^3 + x^2 - 5x + 3$

B1	Show that $(x-1)$ is a factor of $f(x)$.	(2)
B2	Factorise $f(x)$ fully.	(3)
B3	Determine the x and y intercepts of $f(x)$.	(2)
B4	Determine the co-ordinates of the turning point(s) of $f(x)$.	(4)
B5	Find the x-value of the point of inflection of $f(x)$	(1)
B6	Draw a sketch graph of $f(x)$.	(2)
B7	For which value(s) of x is $f(x)$ increasing?	(2)
B8	Describe one transformation of $f(x)$ that, when applied, will result in $f(x)$ having two unequal positive real roots.	(2)
B9	Give the equation of g if g is the reflection of f in the y-axis.	(3)
B10	Determine the average rate of change of f between the points (0:3) and (1:0).	(2)
B11	Determine the equation of the tangent to the f when x = -2.	(4)
B12	Prove that the tangent in B11 will intersect or touch the curve of f at two places.	(4)
		[31]

1C

The tangent to the curve of $g(x) = 2x^3 + px^2 + qx - 7$ at $x = 1$ has the equation $y = 5x - 8$.

C1	Show that (1 ; – 3) is the point of contact of the tangent to the graph.	(1)
C2	Hence or otherwise, calculate the values of p and q.	(6)
		[7]

1D

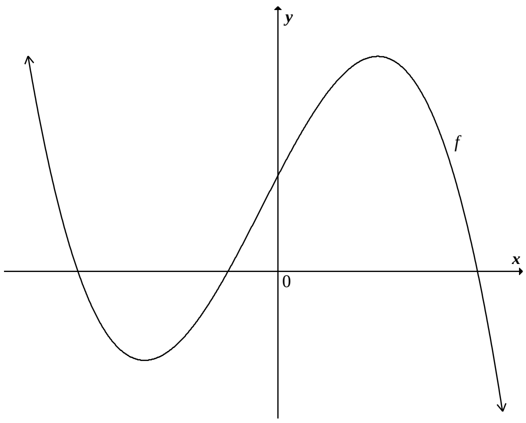
A cubic function f has the following properties:

- $f\left(\frac{1}{2}\right) = f(3) = f(-1) = 0$
- $f'(2) = f'\left(-\frac{1}{3}\right) = 0$
- f decreases for $x \in \left[-\frac{1}{3} ; 2\right]$ only

Draw a possible sketch graph of f, clearly indicating the x-coordinates of the turning points and ALL the x-intercepts. **[4]**

2A

A1 The graph of the function $f(x) = -x^3 - x^2 + 16x + 16$ is sketched below.



A1.1 Calculate the x-coordinates of the turning points of f. (4)

A1.2 Calculate the x-coordinate of the point at which $f'(x)$ is a maximum. (3)

A2 Consider the graph of $g(x) = -2x^2 - 9x + 5$.

A2.1 Determine the equation of the tangent to the graph of g at $x = -1$. (4)

A2.2 For which values of q will the line $y = -5x + q$ not intersect the parabola? (3)

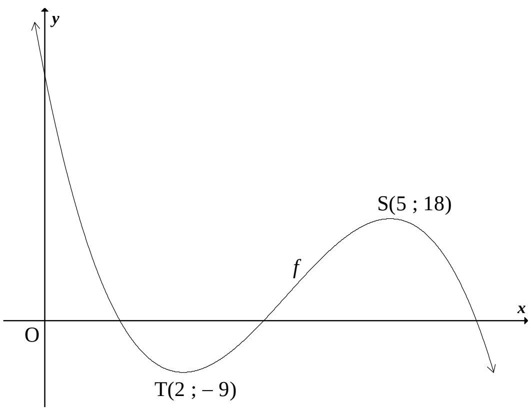
A3 Given: $h(x) = 4x^3 + 5x$

Explain if it is possible to draw a tangent to the graph of h that has a negative gradient. Show ALL your calculations. (3)

[17]

2B

The function $f(x) = -2x^3 + ax^2 + bx + c$ is sketched below. The turning points of the graph of f are T(2 ; – 9) and S(5 ; 18).



B1 Show that $a = 21$, $b = -60$ and $c = 43$. (7)

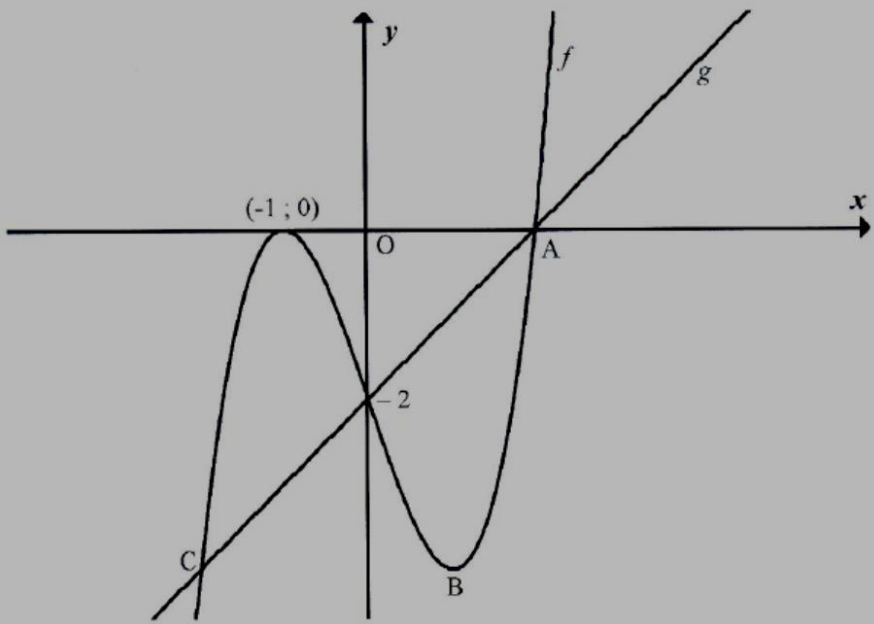
B2 Determine an equation of the tangent to the graph of f at $x = 1$. (5)

B3 Determine the x-value at which the graph of f has a point of inflection. (2)

[14]

2C

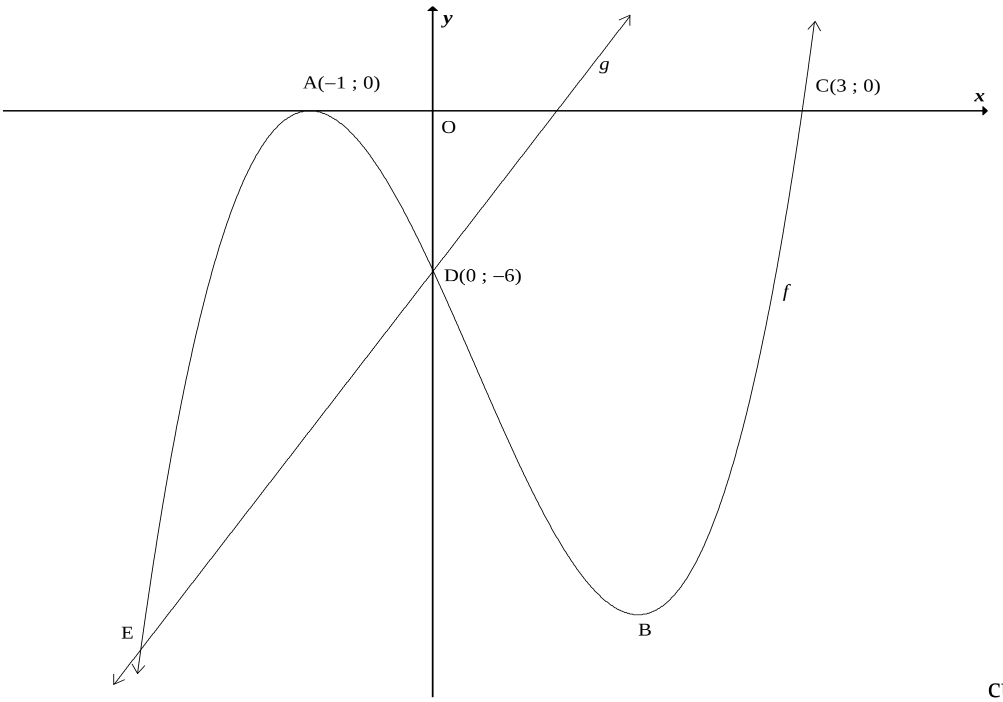
The graph below represents the functions f and g with $f(x) = ax^3 - cx - 2$ and $g(x) = x - 2$.
A and $(-1\ ;\ 0)$ are the x-intercepts of f. The graphs of f and g intersect at A and C.



C1	Determine the coordinates of A.	(1)
C2	Show by calculation that $a = 1$ and $c = -3$.	(4)
C3	Determine the coordinates of B, a turning point of f.	(3)
C4	Show that the line BC is parallel to the x-axis.	(7)
C5	Find the x-coordinate of the point of inflection of f.	(2)
C6	Write down the values of k for which $f(x) = k$ will have only ONE root.	(3)
C7	Write down the values of x for which $f'(x) < 0$.	(2) **[22]**

2D

The graphs of $f(x) = ax^3 + bx^2 + cx + d$ and $g(x) = 6x - 6$ are sketched below.
A(– 1 ; 0) and C(3 ; 0) are the x-intercepts of f.
The graph of f has turning points at A and B.
D(0 ; – 6) is the y-intercept of f.
E and D are points of intersection of the graphs of f and g.



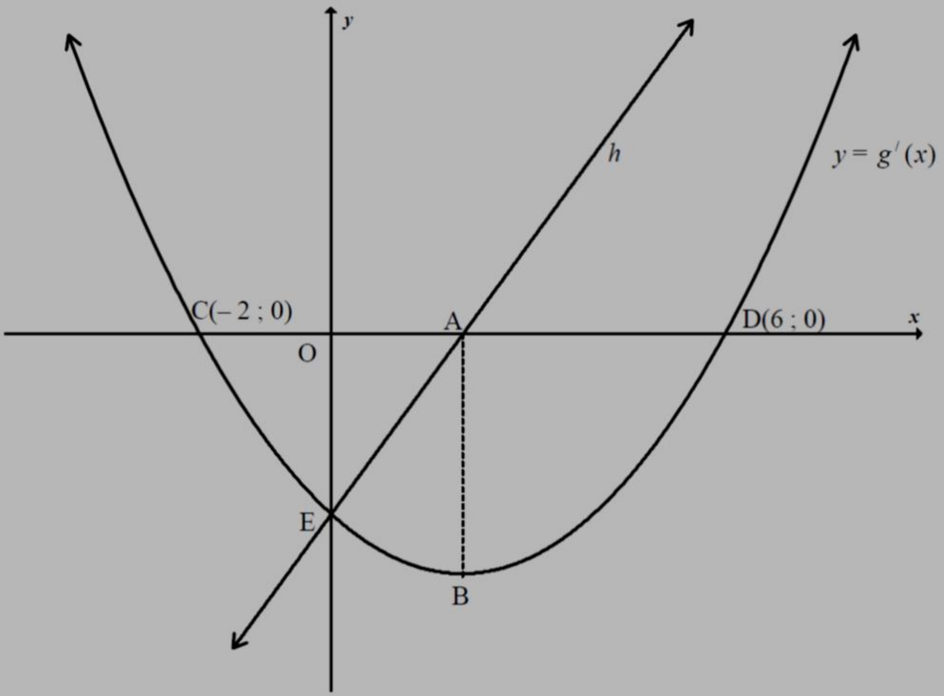
cubic

D1	Show that $a = 2$; $b = -2$; $c = -10$ and $d = -6$.	(5)
D2	Calculate the coordinates of the turning point B.	(5)
D3	$h(x)$ is the vertical distance between $f(x)$ and $g(x)$, that is $h(x) = f(x) - g(x)$. Calculate x such that $h(x)$ is a maximum, where $x < 0$.	(5)
		[15]

3A

The graphs of $y = g'(x) = ax^2 + bx + c$ and $h(x) = 2x - 4$ are sketched below. The graph of $y = g'(x) = ax^2 + bx + c$ is the derivative graph of a cubic function g.

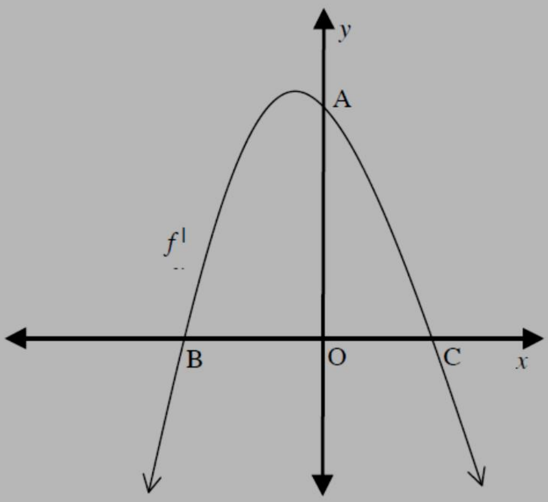
The graphs of h and g' have a common y-intercept at E.
C(– 2 ; 0) and D(6 ; 0) are the x-intercepts of the graph of g'.
A is the x-intercept of h and B is the turning point of g'.
AB || y-axis.



A1	Write down the coordinates of E.	(1)
A2	Determine the equation of the graph of g' in the form $y = ax^2 + bx + c$.	(4)
A3	Write down the x-coordinates of the turning points of g.	(2)
A4	Write down the x-coordinate of the point of inflection of the graph of g.	(2)
A5	Explain why g has a local maximum at $x = -2$.	(3) **[12]**

3B

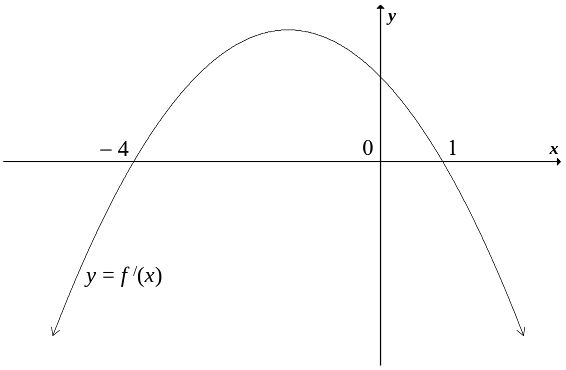
Sketched below is the graph of f', the derivative of $f(x) = -2x^3 - 3x^2 + 12x + 20$. A, B and C are the intercepts of f' with the axes.



B1	Determine the coordinates of A.	(2)
B2	Determine the coordinates of B and C.	(3)
B3	Which points on the graph of $f(x)$ will have exactly the same x-values as B and C?	(1)
B4	For which values of x will $f(x)$ be increasing?	(2)
B5	Determine the y-coordinate of the point of inflection of f.	(4)
		[12]

3C

The graph of $y = f'(x)$, where f is a cubic function, is sketched below.



Use the graph to answer the following questions:

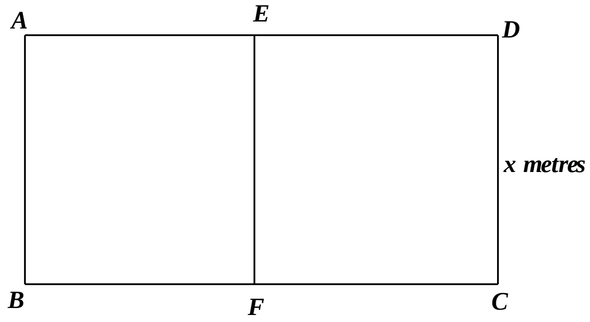
C1 For which values of x is the graph of $y = f'(x)$ decreasing? (1)

C2 At which value of x does the graph of f have a local minimum? Give reasons for your answer. (3)

[4]

Lesson 2: maxima and minima (Optimisation)

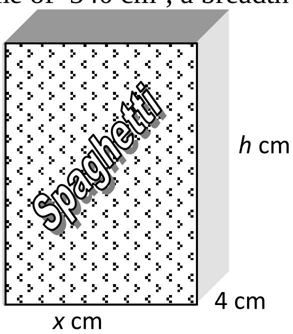
1. The area of .rectangle ABCD sketched below is 2 400 m^2. DC = x *metres*.



1.1 Express AD in terms of x.

1.2 If the rectangle is fenced and the fence EF divides the rectangle in half, find the length of x so that the total length of fencing is minimised.

2. A pasta company has packaged their spaghetti in a box that has the shape of a rectangular prism as shown in the diagram below. The box has a volume of 540 cm^3, a breadth of 4 cm and a length of x cm.



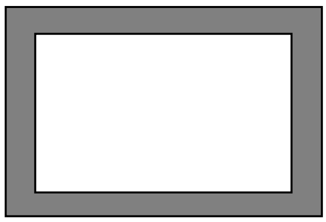
2.1 Express h in terms of x. (2)

2.2 Hence show that the total surface area of the box (in cm^2) is given by:
$A = 8x + 1080x^{-1} + 270$ (3)

2.3 Determine the value of x for which the total surface area is a minimum. Round the answer off to the nearest cm. (4)

[9]

3. A mirror is set into a wooden frame which is 2cm wide.
The outside perimeter of the wooden frame is 72cm.



3.1 The length of the frame is x cm. Determine the breadth of the frame in terms of x. (1)

3.2 Determine the length and breadth of the mirror in terms of x. (2)

3.3 Show that the area of the mirror is given by the function: $A(x) = -x^2 + 36x - 128\ cm^2$ (2)

3.4 Calculate the dimensions of the mirror with the largest area that can fit into the frame. (4)

[9]

Question 4

A wire, 4 metres long, is cut into two pieces. One is bent into the shape of a square and the other into the shape of a circle.

4.1 If the length of wire used to make the circle is x metres, write in terms of x the length of the sides of the square in metres. (1)

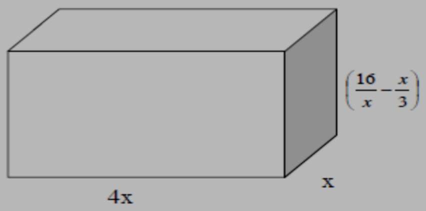
4.2 Show that the sum of the areas of the circle and the square is given by $f(x)=\left(\frac{1}{16}+\frac{1}{4\pi}\right)x^2-\frac{x}{2}+1$ square metres. (4)

4.3 How should the wire be cut so that the sum of the areas of the circle and the square is a minimum? (3)

[8]

Question 5

The base of a rectangular box has dimensions of $4x$ cm and x cm. The height of the box is given as $\left(\frac{16}{x}-\frac{x}{3}\right)$cm.



5.1 Write down an expression for the volume of the box in simplified form. (2)

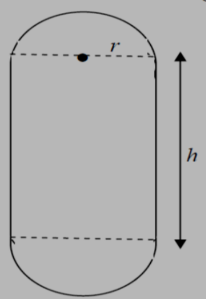
5.2 Calculate the dimensions of the box that would give a maximum volume. (5)

[7]

Question 6

A satellite is to be constructed in the shape of a cylinder with a hemisphere at each end. The radius of the cylinder is r metres and its height is h metres (see diagram below). The outer surface area of the satellite is to be coated with heat-resistant material which is very expensive.

The volume of the satellite has to be $\frac{\pi}{6}$ cubic metres.



Outer surface area of a sphere $= 4\pi r^2$
Curved surface area of a cylinder $= 2\pi rh$
Volume of a sphere $= \frac{4}{3}\pi r^3$
Volume of a cylinder $= \pi r^2 h$

6.1 Show that $h=\frac{1}{6r^2}-\frac{4r}{3}$. (3)

6.2 Hence, show that the outer surface area of the satellite can be given as $S=\frac{4\pi r^2}{3}+\frac{\pi}{3r}$. (3)

6.3 Calculate the minimum outer surface area of the satellite. (6)

[12]

Lesson 3: Rate of change and calculus of motion

1. Water is flowing into a tank at a rate of 5 litres per minute. At the same time water flows out tank at a rate of k litres per minute. The volume (in litres) of water in the tank at time t (in m is given by the formula $V(t) = 100 - 4t$.

1.1 What is the initial volume of the water in the tank? (1)

1.2 Write down TWO different expressions for the rate of change of the volume of water in the tank. (3)

1.3 Determine the value of k (that is, the rate at which water flows out of the tank). (2)

[6]

2. A particle moves along a straight line. The distance, s, (in metres) of the particle from a fixed point on the line at time t seconds ($t \geq 0$) is given by $s(t) = 2t^2 - 18t + 45$.

2.1 Calculate the particle's initial velocity. (Velocity is the rate of change of distance.) (3)

2.2 Determine the rate at which the velocity of the particle is changing at t seconds. (1)

2.3 After how many seconds will the particle be closest to the fixed point? (2)

[6]

3. $s = 5t^2$ is a formula for the distance, s, in metres, fallen by a stone after t seconds if dropped from the top of a cliff .

3.1 How far has the stone fallen after 2 seconds?

3.2 What is the average speed of the stone in the third second (between t=2 and t =3)?

3.3 What will be the instantaneous speed of the stone after 3 seconds?

3.4 When will the instantaneous speed be 20 metres per second?

3.5 How long will it take for the stone to hit the ground if the cliff is 320 metres high?

3.6 What will be the speed of the stone when it hits the ground?

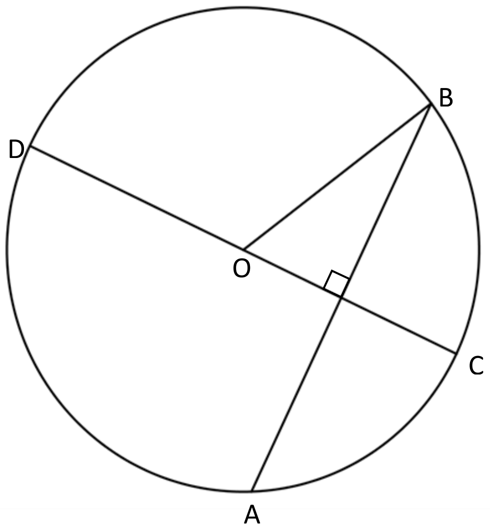
Session 1: Grade 11 geometry

Grade 11 theorems

1. The line drawn from the centre of a circle perpendicular to the chord bisects the chord
2. The perpendicular bisector of a chord passes through the centre of the circle
3. The angle subtended by an arc at the centre of a circle is double the angle subtended by the same arc at the circle (on the same side of the arc as the centre)
4. Angles subtended by an arc or chord of the circle on the same side of the chord are equal
5. The opposite angles of a cyclic quadrilateral are supplementary
6. Two tangents drawn to a circle from the same point outside the circle are equal in length (I**f two tangents to a circle are drawn from a point outside the circle, the distances between this point and the points of contact are equal)**.
7. The angle between the tangent of a circle and the chord drawn from the point of contact is equal to the angle in the alternate segment.

Question 1

In the diagram below, O is the centre of the circle. Chord AB is perpendicular to diameter DC. CM : MD = 4 : 9 and AB = 24 units.



(a) Determine an expression for DC in terms of x if CM = $4x$ units.

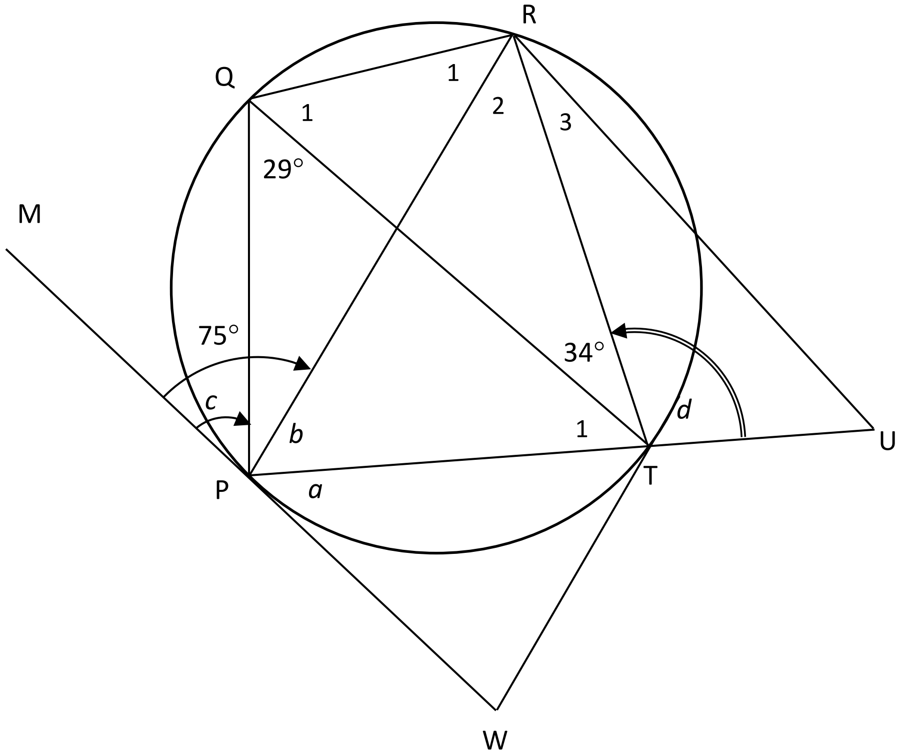
(b) Determine an expression for OM in terms of x.

(c) Hence, or otherwise, calculate the length of the radius.

Question 2

In the diagram points P, Q, R and T lie on the circumference of a circle. MW and TW are tangents to the circle at P and T respectively. PT is produced to meet RU at U. $M\hat{P}R = 75°$ $P\hat{Q}T = 29°$ $Q\hat{T}R = 34°$

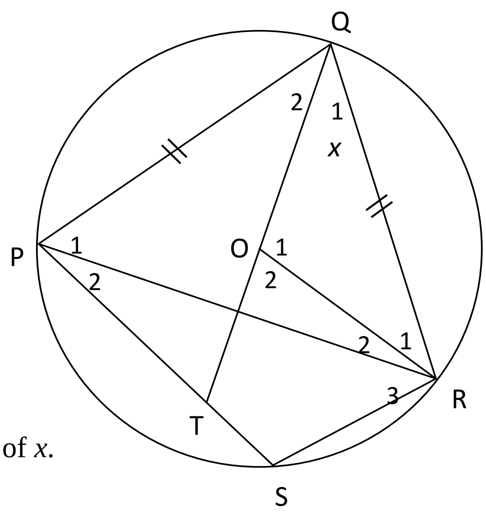
Let $T\hat{P}W = a$, $R\hat{P}T = b$, $M\hat{P}Q = c$ and $R\hat{T}U = d$, calculate the values of *a*, *b*, *c* and *d*.



Question 3

In the diagram below, O is the centre of the circle. P, Q, R and S are points on the circumference of the circle. TOQ is a straight line such that T lies on PS.

PQ = QR and $\hat{Q}_1 = x$.



3.1 Calculate, with reasons, $\hat{P}_1$ in terms of *x*. (3)

3.2 Show that TQ bisects $P\hat{Q}R$. (3)

3.3 Show that STOR is a cyclic quadrilateral. (3)

Session 2

Grade 12 theorems

1. The line drawn parallel to one side of a triangle divides the other two sides proportionally (The midpoint theorem is a special case of this theorem)

2. Equiangular triangles are similar

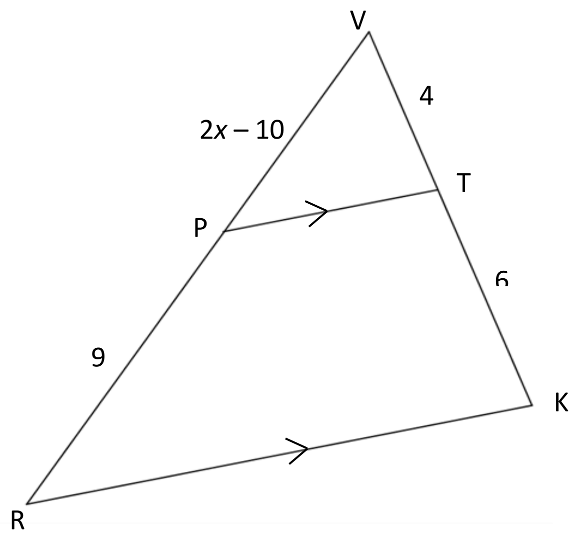
3. Triangles with sides in proportion are similar

4. Prove the Pythagorean theorem by similar triangles

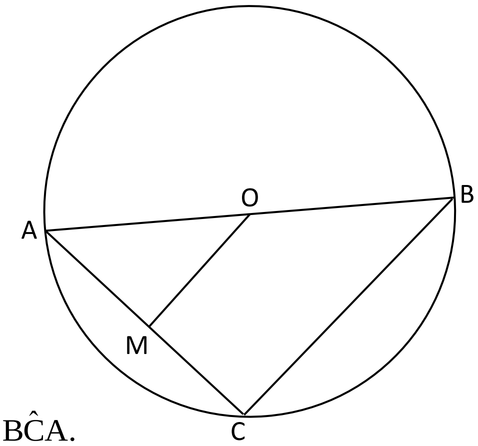
Question 1

1.1 In the diagram below, ΔVRK has P on VR and T on VK such that PT $\parallel$ RK. VT = 4 units, PR = 9 units, TK = 6 units and VP = $2x - 10$ units.

Calculate the value of x.



1.2 O is the centre of the circle below. OM $\perp$ AC. The radius of the circle is equal to 5 cm and BC = 8 cm.



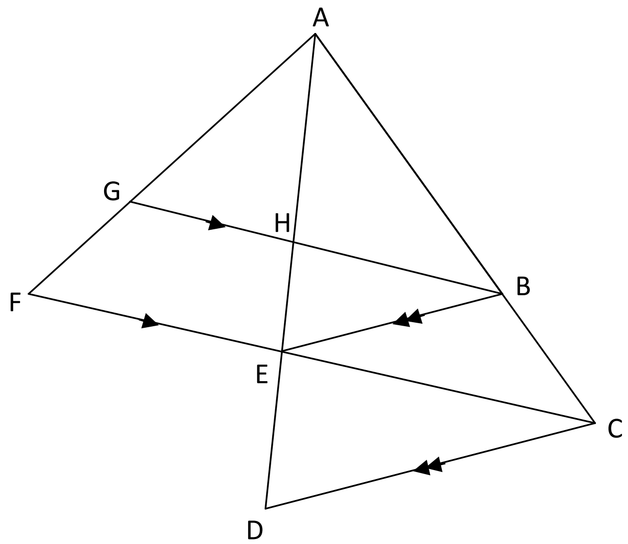
1.2.1 Write down the size of $B\hat{C}A$. (1)

1.2.2 Calculate:

(a) The length of AM, with reasons (3)

(b) Area ΔAOM : Area ΔABC (3)

In the figure below, GB || FC and BE || CD. AC = 6 cm and $\frac{AB}{BC} = 2$.



1.3 Calculate with reasons:

1.3.1 AH : ED (4)

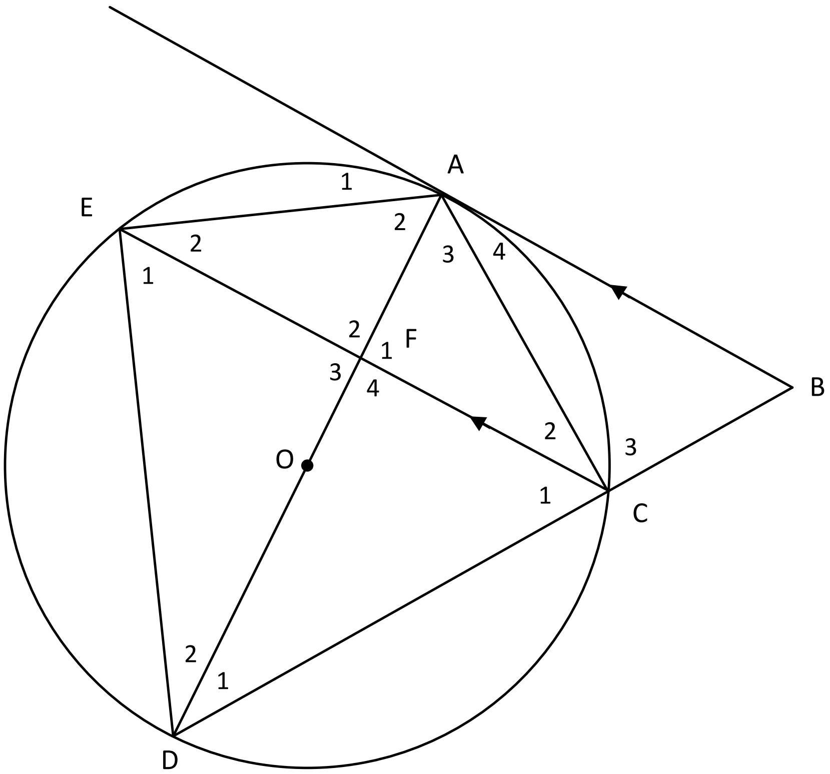
1.3.2 $\frac{BE}{CD}$ (2)

1.3.3 If HE = 2 cm, calculate the value of AD × HE. (2)

[8]

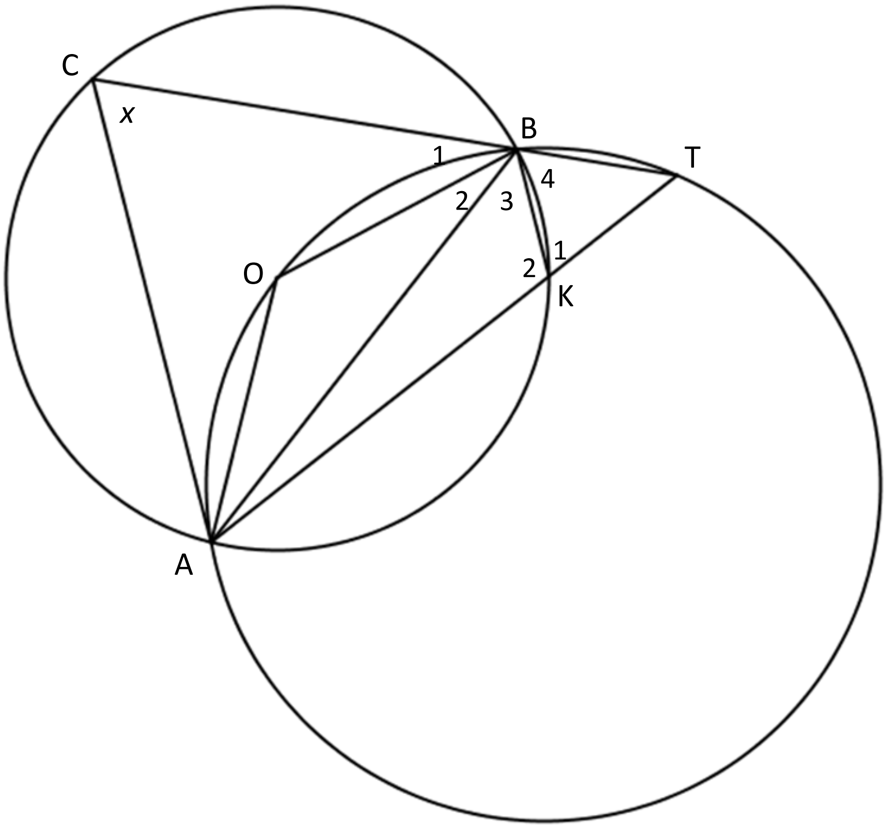
Question 2

1.1 In the figure below, AB is a tangent to the circle with centre O. AC = AO and BA || CE. DC produced, cuts tangent BA at B.



2.1.1	Show $\hat{C}_2 = \hat{D}_1$.	(3)
2.1.2	Prove that ΔACF ||| ΔADC.	(3)
2.1.3	Prove that AD = 4AF.	(4)

2.2.O is the centre of the circle CAKB. AK produced intersects circle AOBT at T. $A\hat{C}B = x$



2.2.1 Prove that $\hat{T} = 180° - 2x$. (3)

2.2.2 Prove AC || KB. (5)

2.2.3 Prove ΔBKT ||| ΔCAT (3)

2.2.4 If AK : KT = 5 : 2, determine the value of $\frac{AC}{KB}$ (3)

[14]